Z-mat training

Z-set group

Mines ParisTech, CNRS UMR 7633
Centre des Matériaux
BP 87, 91003 Evry cedex, France

www.zset-software.com

3D plasticity and viscoplasticity

- Strain partition

$$\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + \underset{\sim}{\epsilon}^{th} + \underset{\sim}{\epsilon}^{p} + \underset{\sim}{\epsilon}^{vp}$$

$$\underset{\sim}{\sigma} = \underset{\approx}{\Lambda} : \underset{\sim}{\epsilon}^{el}$$

$$\underset{\sim}{\epsilon}^{th} = (T - T_I)\,\underset{\sim}{\alpha}$$

- Criterion

$$f$$

- Flow rule

$$\dot{\underset{\sim}{\epsilon}}^{p} = ...$$

- Hardening rule

$$\dot{Y}_I = ...$$

Formulation of viscoplastic constitutive equations

The easiest way of writing a viscoplastic model is to define a *viscoplastic potential*, Φ, depending on stress and hardening variables. A *standard* model will then be characterized using the yield function f to define Φ, and deriving viscoplastic strain rate and hardening rate from Φ, $\Phi := \Phi(f(\underset{\sim}{\boldsymbol{\sigma}}, Y_I))$.

- Viscoplastic strain rate:

$$\dot{\underset{\sim}{\boldsymbol{\epsilon}}}^{vp} = \frac{\partial \Phi}{\partial \underset{\sim}{\boldsymbol{\sigma}}}$$

- State variable rate:

$$\dot{\alpha}_I = -\frac{\partial \Phi}{\partial Y_I}$$

Introducing $\dot{v} = \dfrac{\partial \Phi}{\partial f}$, $\underset{\sim}{\boldsymbol{n}} = \partial f / \partial \underset{\sim}{\boldsymbol{\sigma}}$, and $M_I = \partial f / \partial Y_I$

$$\dot{\underset{\sim}{\boldsymbol{\epsilon}}}^{vp} = \dot{v}\,\underset{\sim}{\boldsymbol{n}} \qquad \dot{\alpha}_I = -\dot{v}\, M_I$$

Examples of simple viscoplastic models

- Norton rule and von Mises criterion $f = J(\underset{\sim}{\sigma})$, and :

$$\Phi = \frac{K}{n+1}\left(\frac{J(\underset{\sim}{\sigma})}{K}\right)^{n+1}$$

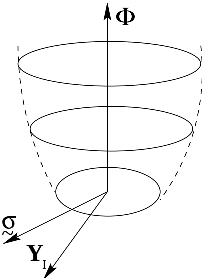
$$\dot{\underset{\sim}{\epsilon}}^{vp} = \left(\frac{J}{K}\right)^{n}\frac{\partial J}{\partial \underset{\sim}{\sigma}}$$

$$\frac{\partial J}{\partial \underset{\sim}{\sigma}} = \frac{\partial J}{\partial \underset{\sim}{\boldsymbol{s}}} : \frac{\partial \underset{\sim}{\boldsymbol{s}}}{\partial \underset{\sim}{\sigma}} = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}}}{J} : (\underset{\approx}{\boldsymbol{I}} - \frac{1}{3}\underset{\sim}{\boldsymbol{I}} \otimes \underset{\sim}{\boldsymbol{I}}) = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}}}{J}$$

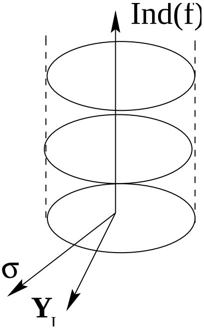
  The elastic domain is reduced to one point.
- Bingham model:

$$\Phi = \frac{1}{2}\left(\frac{J(\underset{\sim}{\sigma}) - \sigma_y}{\eta}\right)^{2}$$

From viscoplasticity to plasticity



a. Viscoplastic potential



b. Plastic pseudo-potential as a limit case

- *Viscoplasticity* = after the choice of the function defining viscous effect, $\dot{v}$ is known
- *Plasticity* = $\dot{\lambda}$ to be defined from the consistency condition

Formulation of the plastic constitutive equations

- elastic domain : $f(\underset{\sim}{\sigma}, Y_I) < 0$ $\quad (\dot{\underset{\sim}{\epsilon}} = \underset{\approx}{\Lambda}^{-1} : \dot{\underset{\sim}{\sigma}})$
- elastic unloading : $f(\underset{\sim}{\sigma}, Y_I) = 0$ and $\dot{f}(\underset{\sim}{\sigma}, Y_I) < 0$ $(\dot{\underset{\sim}{\epsilon}} = \underset{\approx}{\Lambda}^{-1} : \dot{\underset{\sim}{\sigma}})$
- plastic flow : $f(\underset{\sim}{\sigma}, Y_I) = 0$ and $\dot{f}(\underset{\sim}{\sigma}, Y_I) = 0$ $(\dot{\underset{\sim}{\epsilon}} = \underset{\approx}{\Lambda}^{-1} : \dot{\underset{\sim}{\sigma}} + \dot{\underset{\sim}{\epsilon}}^p)$

$$\dot{\underset{\sim}{\epsilon}}^p = ...$$

$$\dot{Y}_I = ...$$

Flow directions associated with von Mises criterion

$f(\underset{\sim}{\boldsymbol{\sigma}}) = J(\underset{\sim}{\boldsymbol{\sigma}}) - \sigma_y$ (no hardening)

$$\underset{\sim}{\boldsymbol{n}} = \frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} = \frac{\partial J}{\partial \underset{\sim}{\boldsymbol{\sigma}}} = \frac{\partial J}{\partial \underset{\sim}{\boldsymbol{s}}} : \frac{\partial \underset{\sim}{\boldsymbol{s}}}{\partial \underset{\sim}{\boldsymbol{\sigma}}} \quad \text{where :} \quad n_{ij} = \frac{\partial J}{\partial s_{kl}} \frac{\partial s_{kl}}{\partial \sigma_{ij}}$$

$$\frac{\partial s_{kl}}{\partial \sigma_{ij}} = \delta_{ik}\,\delta_{jl} - \frac{1}{3}\,\delta_{ij}\,\delta_{kl}$$

$$n_{ij} = \frac{3}{2}\frac{s_{ij}}{J} \quad \text{where:} \quad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}}}{J}$$

Pure tension along direction 1 :

$$\underset{\sim}{\boldsymbol{s}} = \frac{2\,\sigma}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} \; ; \; J = |\sigma| \; ; \; \underset{\sim}{\boldsymbol{n}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} sign(\sigma)$$

Prandtl-Reuss law (1)

$$f(\underset{\sim}{\sigma}, R) = J(\underset{\sim}{\sigma}) - \sigma_y - R(p)$$

- Hardening curve for one-dimensional monotonic loading:
$\sigma = \sigma_y + R(p)$.
- Plastic modulus: $H = dR/d\epsilon^p = dR/dp$

For pure tension:

$$n_{11} = sign(\sigma) \quad , \quad n_{22} = n_{33} = (-1/2)n_{11}$$

$$\dot{\epsilon}^p_{11} = \dot{\epsilon}^p = sign(\sigma)\dot{\lambda} \quad , \quad \dot{\epsilon}_{22} = \dot{\epsilon}_{33} = (-1/2)\dot{\epsilon}^p$$

$$\dot{p} = |\dot{\epsilon}^p| = \dot{\lambda}$$

For general 3D case:

$$\dot{\underset{\sim}{\epsilon}}^p : \dot{\underset{\sim}{\epsilon}}^p = \dot{\lambda}^2 \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\dot{\lambda} \quad \text{then } \dot{p} = \left(\frac{2}{3}\dot{\underset{\sim}{\epsilon}}^p : \dot{\underset{\sim}{\epsilon}}^p\right)^{1/2}$$

Prandtl-Reuss law (2)

- Use of the consistency condition:

$$\frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} : \underset{\sim}{\dot{\boldsymbol{\sigma}}} + \frac{\partial f}{\partial R}\dot{R} = 0 \quad \text{writes:} \quad \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\dot{\boldsymbol{\sigma}}} - H\dot{p} = 0 \quad \text{and:}$$

$$\dot{\lambda} = \frac{\underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\dot{\boldsymbol{\sigma}}}}{H} \text{ with } \qquad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}}}{J}$$

$$\underset{\sim}{\dot{\boldsymbol{\epsilon}}}^p = \dot{\lambda}\,\underset{\sim}{\boldsymbol{n}} = \frac{\underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\dot{\boldsymbol{\sigma}}}}{H}\,\underset{\sim}{\boldsymbol{n}}$$

For pure tension:

$$n_{11} = sign(\sigma) \quad , \quad \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\dot{\boldsymbol{\sigma}}} = \dot{\sigma}\, sign(\sigma) \quad \text{and:} \quad \dot{\lambda} = \dot{p} = \dot{\epsilon}^p_{11}$$

$$\text{so that :} \quad \dot{\epsilon}^p = \frac{n_{11}\,\dot{\sigma}}{H}\, n_{11} = \frac{\dot{\sigma}}{H}$$

Prager rule (1)

$$f(\underset{\sim}{\boldsymbol{\sigma}}, \underset{\sim}{\boldsymbol{X}}) = J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}) - \sigma_y \quad \text{with} \quad J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}) = \left((3/2)(\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}) : (\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}})\right)^{0,5}$$

One-dimensional loading :

$$|\sigma - X| - \sigma_y = 0$$

Tensile curve modeled by:

$$\sigma = X(\epsilon^p) + \sigma_y$$

Since $\underset{\sim}{\boldsymbol{X}}$ is proportional to $\underset{\sim}{\boldsymbol{\epsilon}}^p$, its components for one-dimensional loading are

$$X_{11}, X_{22} = X_{33} = -(1/2)X_{11}$$

Let us define:

$$\underset{\sim}{\boldsymbol{X}} = (2/3)H\underset{\sim}{\boldsymbol{\epsilon}}^p$$

For one-dimensional loading, assume:

$$X = (3/2)X_{11} = H\epsilon^p_{11}$$

then

$$\begin{aligned} \underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}} &= diag\left((2/3)\sigma - X_{11}, -(1/3)\sigma + X_{11}/2, id\right) \\ &= diag\left((2/3)(\sigma - X), -(1/3)(\sigma - X), id\right) \end{aligned}$$

$$J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}) = |\sigma - X|$$

Prager rule (2)

Consistency condition:

$$\frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} + \frac{\partial f}{\partial \underset{\sim}{\boldsymbol{X}}} : \dot{\underset{\sim}{\boldsymbol{X}}} = 0 \quad \text{then :} \quad \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} - \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{X}}} = 0 \quad \text{with :} \quad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2} \frac{\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}}{J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}})}$$

$$\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} = \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{X}}} = \underset{\sim}{\boldsymbol{n}} : \frac{2}{3} H \dot{\lambda} \, \underset{\sim}{\boldsymbol{n}} = H \dot{\lambda} \quad \text{so that :} \quad \dot{\lambda} = (\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}})/H$$

$$\dot{\underset{\sim}{\epsilon}}^p = \dot{\lambda} \, \underset{\sim}{\boldsymbol{n}} = \frac{\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}}}{H} \underset{\sim}{\boldsymbol{n}}$$

- Same formal expression than for isotropic hardening, nevertheless $\underset{\sim}{\boldsymbol{n}}$ is different;
- Under one-dimensional loading, $\sigma = \sigma_{11}$, with $X = (3/2)X_{11}$:

$$|\sigma - X| = \sigma_y \quad , \quad \dot{\sigma} = \dot{X} = H \dot{\epsilon}^p$$

Summary in plasticity and viscoplasticity

For both cases:

- elastic domain defined by the load function $f < 0$;
- isotropic and kinematic hardenings

For plastic materials:

- plastic flow defined by the consistency condition, $f = 0, \dot{f} = 0$;
- plastic flow is *time independent* :

$$d\varepsilon^p = g(\sigma, \ldots)d\sigma$$

For viscoplastic materials:

- viscoplastic flow is defined by the value of the overstress $f > 0$;
- possible hardening on the viscous stress;
- viscoplastic flow if *time dependent* :

$$d\varepsilon^{vp} = g(\sigma, \ldots)dt$$

State variables

- Isotropic hardening depend on **p**, the *accumulated plastic strain* defined as :

$$\dot{p} = |\dot{\epsilon}^p|$$

- Linear kinematic hardening depend on ϵ^p , the *present plastic strain*

- Nonlinear kinematic hardening depend on α , defined as :

$$\dot{\alpha} = (\ 1 - D\ \alpha\ sign(\dot{\epsilon}^p)\ )\ \dot{\varepsilon}^p$$

asymptotic value of α *= 1 / D*

Isotropic/Kinematic hardening

$$f\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}\right) - R$$

- **Non-linear isotropic hardening**

  $R = R0 + Q\ \left(1\ -\ e^{-bp}\right)$

  saturation rate: b , saturation hardening: Q

- **Non-linear kinematic hardening**

  $\underset{\sim}{\boldsymbol{X}} = \frac{2}{3} C \underset{\sim}{\alpha} \quad , \quad \dot{\underset{\sim}{\alpha}} = \dot{p}\left[\underset{\sim}{\boldsymbol{n}} - \frac{3D}{2C}\underset{\sim}{\boldsymbol{X}}\right]$

  saturation rate: D , saturation hardening: $\frac{C}{D}$

demo reset open terminal

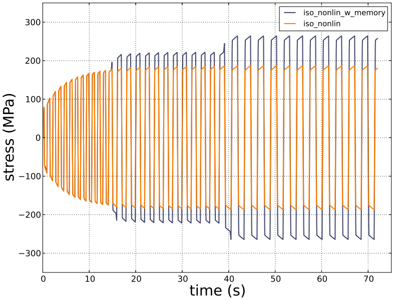
Cyclic plasticity and hardening models

- kinematic hardening : shape of the stress-strain loops
- isotropic hardening :
  cyclic hardening ($Q > 0$) or softening ($Q < 0$)

▸ demo ▸ reset ▸ open terminal

Strain-range memory effect

- Non linear isotropic hardening with strain-range memory:



$$Q = Q_0 + (Q_{sat} - Q_0)\exp(-2\mu q)$$
$$R = R_0 + Q\left(1 - e^{-\mathbf{b}p}\right)$$

$$\underset{\sim}{\boldsymbol{n}}^* = \frac{1}{2}(\underset{\sim}{\boldsymbol{\epsilon_{vi}}} - \underset{\sim}{\boldsymbol{z}})/J(\underset{\sim}{\boldsymbol{\epsilon_{vi}}} - \underset{\sim}{\boldsymbol{z}})$$
$$\eta = \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\boldsymbol{n}}^*$$
$$\dot{q} = \eta\dot{\lambda}$$

$$\dot{\underset{\sim}{\boldsymbol{z}}} = 2(\underset{\sim}{\boldsymbol{n}}^* : \underset{\sim}{\boldsymbol{\epsilon_{vi}}})\underset{\sim}{\boldsymbol{n}}^*$$

▸ demo ▸ reset ▸ open terminal

Multi-kinematic models

$$f\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \sum \underset{\sim}{\boldsymbol{X}}_i\right) - R$$

- smooth out the transition from linear to nonlinear behavior
- modelling of short and long-range hardening mechanisms
- for model calibration :
  - fix D values to scan the saturation rates :
    $D1 = 200$, $D2 = 4\ D1$, $D3 = 4\ D2$ etc...
  - find the C values using optimization

demo reset open terminal

Asymmetric strain-controlled cyclic test

- linear kinematic hardening : non-zero mean-stress
- non-linear kinematic hardening : mean-stress relaxation to zero
- linear + non-linear : mean-stress relaxation to non-zero value

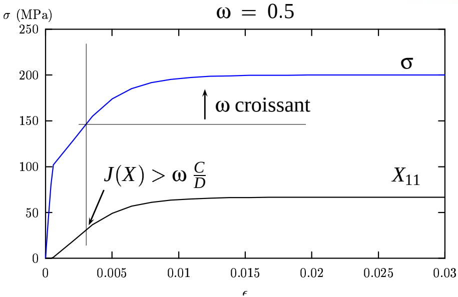
demo reset open terminal

Asymmetric stress-controlled cyclic test

- linear kinematic hardening : no ratcheting
- non-linear kinematic hardening :
  possible ratcheting with a constant strain increase
- linear + non-linear : arrest of the ratcheting effect

demo reset open terminal

Non-linear kinematic with threshold



$$\underset{\sim}{X} = \frac{2}{3} C \underset{\sim}{\alpha}$$

$$\dot{\underset{\sim}{\alpha}} = \left[\underset{\sim}{n} - \frac{3D}{2C} \underset{\approx}{\Phi} : \underset{\sim}{X} \right] \dot{p}$$

$$\underset{\approx}{\Phi} = \left\langle \frac{D\, J(\underset{\sim}{X}) - \omega\, C}{1 - \omega} \right\rangle \frac{1}{(D\, J(\underset{\sim}{X}))} \underset{\approx}{I}$$

- switches from linear to nonlinear when $J(\underset{\sim}{X}) > \frac{C}{D}$
- refined modelling of mean-stress relaxation and ratcheting effects
- several objects may be needed to avoid slope discontinuities

▸ demo ▸ reset ▸ open terminal

Viscoplasticity

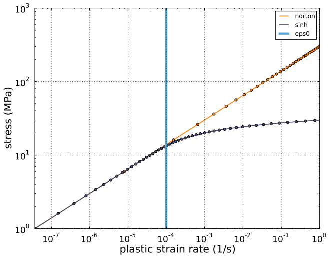
$$
\begin{aligned}
&\textbf{Strain partition} : && \underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + \left(\underset{\sim}{\epsilon}^{th}\right) + \underset{\sim}{\epsilon}^{v} \\
&\textbf{Normality rule} : && \dot{\underset{\sim}{\epsilon}}^{v} = \dot{\lambda}\,\frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} \\
&\textbf{Norton flow law} : && \dot{\lambda} = \left(\frac{\langle f(\underset{\sim}{\boldsymbol{\sigma}})\rangle}{K}\right)^{n}
\end{aligned}
$$

- strain rate sensitivity
- creep
- stress relaxation

▸ demo ▸ reset ▸ open terminal

Norton vs Hyperbolic sine

- Hyperbolic sine function often used for more accurate correlation of strain rate sensitivity over a wide range of inelastic strain rates



$$\dot{p} = \left\langle \frac{\sigma_\upsilon}{K_n} \right\rangle^{n_n}$$ - Norton power low

$$\dot{p} = \epsilon_0 \left[\sinh\left(\left\langle \frac{\sigma_\upsilon}{K_h} \right\rangle^{n_h}\right)\right]^m$$ - Hyperbolic law

same response for both models for low plastic deformation rates

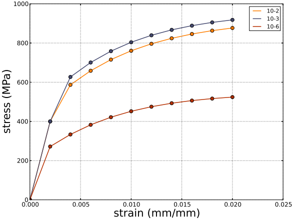
- Coefficient ϵ_0 defines strain rate at which hyperbolic law deviates from the classical norton response
- In order to adjust the coefficient K_h for $\sigma_\upsilon \to 0$ a simple rule can be applied:

suppose $n_n = n_h = n$ **and** $m = 1$, **then** $K_h = \epsilon_0^{\frac{1}{n}} K_n$

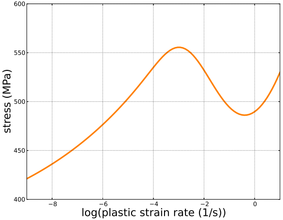
▸ demo ▸ reset ▸ open terminal

Inverse strain rate sensitivity (Portevin-Le Chatelier effect)

- Effect is observed for many metallic materials in some temperature and strain rate domains
- Associated with dynamic strain aging (DSA)



Inverse strain rate sensitivity



Evolutions with plastic strain rate of the normal viscous stress

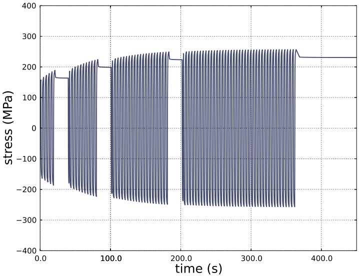
$$R = R_{iso} + R_a \qquad R_a = P1\left(1 - e^{-\left(\frac{t_a}{t_0}\right)^{\beta}}\right) \qquad \dot{t}_a = 1 - \frac{t_a\dot{p}}{w}$$

▸ demo ▸ reset ▸ open terminal

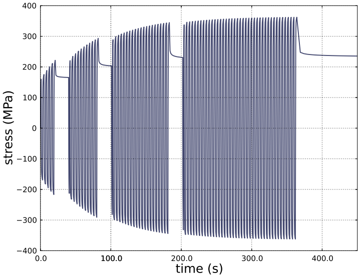
Marquis effect on K

- Influence of cyclic hardening on the viscous part of the stress

$$\sigma_v = K\dot{p}^{1/n} \qquad K = K_0 + \xi R$$



$\xi = 0$

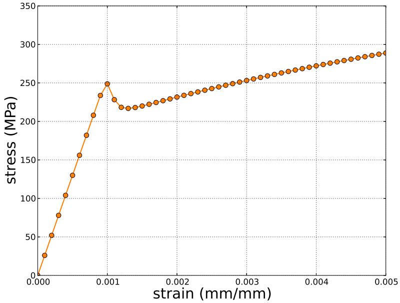


$\xi = 4$

demo reset open terminal

Static Strain Aging

- Same constitutive model for both SSA and DSA
- Possible application: simulation of the peak in the stress-strain curve around the yield strength



$$R = R_{iso} + R_a$$

$$R_a = P1\left(1 - e^{-\left(\frac{t_a}{t_0}\right)^{\beta}}\right)$$

$$\dot{t}_a = 1 - \frac{t_a \dot{p}}{w}$$

demo reset open terminal

Yield surface evolutions

- Z-sim option to draw yield surface evolutions

```
****simulate
 ***test creep
  **load *segment 5
    time sig11 sig22 sig33 sig12
    0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
    1.000000e-03 0.000000e+00 2.000000e+02 0.000000e+00 0.000000e+00
    1.000000e+03 0.000000e+00 2.000000e+02 0.000000e+00 0.000000e+00
  **model ...
  **yield_surface yield_11_22_0
   *degrees 5.0
   *factor 1000.0
   *find_offset
   *component sig11 sig22
   *time 1000.0
****return
```

demo reset open terminal

Kinematic hardening with static recovery

- time-dependent recovery of the hardening at high temperature

$$\dot{\underset{\sim}{\alpha}} = \dot{\lambda} \left[\underset{\sim}{\boldsymbol{n}} - \frac{3}{2}\frac{D}{C} \underset{\sim}{\boldsymbol{X}} \right] - \frac{3}{2} \frac{\underset{\sim}{\boldsymbol{X}}}{J(\underset{\sim}{\boldsymbol{X}})} \left(\frac{J(\underset{\sim}{\boldsymbol{X}})}{M} \right)^m$$

- constant strain rate during creep tests due to a balance of defect creations (hardening) and destruction (recovery)

▸ demo ▸ reset ▸ open terminal

Multi potential models

Strain partition : $\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + (\underset{\sim}{\epsilon}^{th}) + \sum \underset{\sim}{\epsilon}^{i}$

- refined modelling of a wide range of strain rates associated with different deformation mechanisms
- each potential can have its own flow law and hardening objects

▸ demo ▸ reset ▸ open terminal

Interaction between potentials

Example: Two inelastic deformations, one plastic, the other one viscoplastic

Strain partition :

$$\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + \underset{\sim}{\epsilon}^{p} + \underset{\sim}{\epsilon}^{v}$$
$$f^p\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}^p\right) - R^p$$
$$f^v\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}^v\right) - R^v$$

Hardening :

$$\underset{\sim}{\boldsymbol{X}}^p = \frac{2}{3} C_p \underset{\sim}{\alpha}^p + C_{vp} \underset{\sim}{\alpha}^v$$
$$\dot{\underset{\sim}{\alpha}}^p = \dot{p}\left[\underset{\sim}{\boldsymbol{n}}^p - \frac{3D}{2C} \underset{\sim}{\boldsymbol{X}}^p\right]$$
$$\underset{\sim}{\boldsymbol{X}}^v = \frac{2}{3} C_v \underset{\sim}{\alpha}^v + C_{vp} \underset{\sim}{\alpha}^p$$
$$\dot{\underset{\sim}{\alpha}}^v = \dot{\lambda}\left[\underset{\sim}{\boldsymbol{n}}^v - \frac{3D}{2C} \underset{\sim}{\boldsymbol{X}}^v\right]$$

- alternative way to model inverse strain rate effect : Portevin-Le Chatelier effect in austenitic stainless steels

▸ demo ▸ reset ▸ open terminal

2M1C model

- 2 mechanisms, 1 criterion
- allows to control the amount of ratcheting

Criterion : $f = \sqrt{J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}_1)^2 + J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}_2)^2} - R$

Kinematic hardening with coupling term :

$$\underset{\sim}{\boldsymbol{X}}_1 = \frac{2}{3}\left(C_{11}\,\underset{\sim}{\alpha}_1 + C_{12}\,\underset{\sim}{\alpha}_2\right)$$
$$\underset{\sim}{\boldsymbol{X}}_2 = \frac{2}{3}\left(C_{22}\,\underset{\sim}{\alpha}_2 + C_{12}\,\underset{\sim}{\alpha}_1\right)$$

Kinematic evolution : $\dot{\underset{\sim}{\alpha}}_i = \dot{\lambda}\left(\underset{\sim}{\boldsymbol{n}}_i - \frac{3D_i}{2C_{ii}}\underset{\sim}{\boldsymbol{X}}_i\right)$

with : $\underset{\sim}{\boldsymbol{n}}_i = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}_i}{J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}_i)}$

demo reset open terminal

Addition of damage objects

Example: Viscoplastic damage (stage III creep)

Effective stress : $\underset{\sim}{\boldsymbol{\sigma}} = (1-d)\, \underset{\approx}{\boldsymbol{D}}_{el} : \underset{\sim}{\epsilon}_{el} = (1-d)\, \underset{\sim}{\boldsymbol{\sigma}}_{eff}$

Damage evolution using the Hayhurst function :

$$\dot{d} = \left\langle \frac{\chi(\underset{\sim}{\boldsymbol{\sigma}})}{A} \right\rangle^{r} (1-d)^{-k}$$

$$\chi\left(\underset{\sim}{\boldsymbol{\sigma}}\right) = \alpha\, \sigma_{I} + \beta\, tra(\underset{\sim}{\boldsymbol{\sigma}}) + (1-\alpha-\beta) J$$

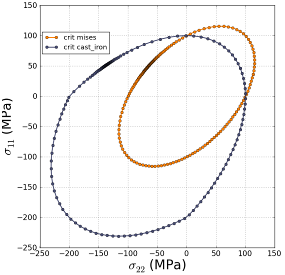
▸ demo ▸ reset ▸ open terminal

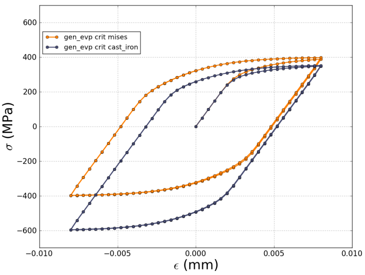
Cast Iron behavior

Ingredients of the cast iron model:

- Modified criterion:

$$f_t = \left(J^2 + (R_c - R_t)\,Tr\,\underset{\sim}{\sigma}\right)^{\frac{1}{2}} - (R_t R_c)^{\frac{1}{2}}$$
$$f_c = J - R_c$$





demo | reset | open terminal

Single-crystal model

Plastic deformation : $\dot{\underset{\sim}{\epsilon}}^p = \sum_r \dot{\gamma}^r \, \underset{\sim}{\boldsymbol{m}}^r$

for each slip system r : orientation tensor $\underset{\sim}{\boldsymbol{m}}^r$, slip γ^r

Resolved shear stress : $\tau^r = \underset{\sim}{\boldsymbol{\sigma}} : \underset{\sim}{\boldsymbol{m}}^r$

Viscoplastic flow : $\dot{v}^r = \left\langle \frac{f^r}{K} \right\rangle^n \quad , \quad \dot{\gamma}^r = \dot{v}^r \, sign(\tau^r - x)$

Criterion : $f^r = |\tau^r - x^r| - r^r - \tau_0$

Kinematic hardening :

$$x^r = C \alpha^r$$

$$\dot{\alpha}^r = (sign(\tau^r - x^r) - D \alpha^r) \, \dot{v}^r$$

Isotropic hardening :

$$r^r = Q \sum_s h_{rs} \, (1 - exp(-b \, v^s))$$

h_{rs} : interaction matrix

demo reset open terminal

Polycrystal model

- Micro-mechanical crystallographic model
- G grain phases defined by their volume fraction f_g and orientation
- Localization rule for stress $\underset{\sim}{\sigma}_g$ in each grain :

$$\underset{\sim}{\sigma}_g = \underset{\sim}{\Sigma} + C \left[\sum_{i \in G} (f_i \beta_i) - \underset{\sim}{\beta}_g \right]$$

- Evolution of inter-granular hardening tensors $\underset{\sim}{\beta}_g$:

$$\dot{\underset{\sim}{\beta}}_g = \dot{\underset{\sim}{\epsilon}}^p_g - D \, \underset{\sim}{\beta}_g \, ||\dot{\underset{\sim}{\epsilon}}^p_g||$$

- Homogenization of plastic strains : $\dot{\underset{\sim}{E}}^p = \sum_g f_g \, \dot{\underset{\sim}{\epsilon}}^p_g$
- Single-crystal constitutive equations for each phase g

demo reset open terminal