Z-mat training

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3D plasticity and viscoplasticity

- Strain partition

$$\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + \underset{\sim}{\epsilon}^{th} + \underset{\sim}{\epsilon}^{p} + \underset{\sim}{\epsilon}^{vp}$$

$$\underset{\sim}{\sigma} = \Lambda : \underset{\sim}{\epsilon}^{el}$$

$$\underset{\sim}{\epsilon}^{th} = (T - T_I)\,\underset{\sim}{\alpha}$$

- Criterion

$$f$$

- Flow rule

$$\dot{\underset{\sim}{\epsilon}}^{p} = \ldots$$

- Hardening rule

$$\dot{Y}_I = \ldots$$

Formulation of viscoplastic constitutive equations

The easiest way of writing a viscoplastic model is to define a *viscoplastic potential*, Φ, depending on stress and hardening variables. A *standard* model will then be characterized using the yield function f to define Φ, and deriving viscoplastic strain rate and hardening rate from Φ, $\Phi := \Phi(f(\underset{\sim}{\sigma}, Y_I))$.

- Viscoplastic strain rate:

$$\dot{\underset{\sim}{\epsilon}}^{vp} = \frac{\partial \Phi}{\partial \underset{\sim}{\sigma}}$$

- State variable rate:

$$\dot{\alpha}_I = -\frac{\partial \Phi}{\partial Y_I}$$

Introducing $\dot{v} = \dfrac{\partial \Phi}{\partial f}$, $\underset{\sim}{n} = \partial f / \partial \underset{\sim}{\sigma}$, and $M_I = \partial f / \partial Y_I$

$$\dot{\underset{\sim}{\epsilon}}^{vp} = \dot{v}\, \underset{\sim}{n} \qquad \dot{\alpha}_I = -\dot{v}\, M_I$$

Examples of simple viscoplastic models

- Norton rule and von Mises criterion $f = J(\underset{\sim}{\sigma})$, and :

$$\Phi = \frac{K}{n+1}\left(\frac{J(\underset{\sim}{\sigma})}{K}\right)^{n+1}$$

$$\underset{\sim}{\dot{\epsilon}}^{vp} = \left(\frac{J}{K}\right)^{n}\frac{\partial J}{\partial \underset{\sim}{\sigma}}$$

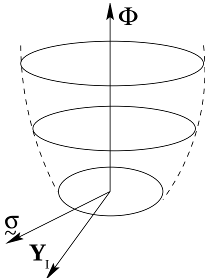
$$\frac{\partial J}{\partial \underset{\sim}{\sigma}} = \frac{\partial J}{\partial \underset{\sim}{s}} : \frac{\partial \underset{\sim}{s}}{\partial \underset{\sim}{\sigma}} = \frac{3}{2}\frac{\underset{\sim}{s}}{J} : (\underset{\approx}{I} - \frac{1}{3}\underset{\sim}{I}\otimes\underset{\sim}{I}) = \frac{3}{2}\frac{\underset{\sim}{s}}{J}$$

  The elastic domain is reduced to one point.

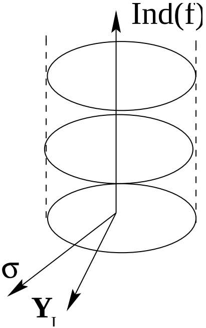
- Bingham model:

$$\Phi = \frac{1}{2}\left(\frac{J(\underset{\sim}{\sigma}) - \sigma_y}{\eta}\right)^{2}$$

From viscoplasticity to plasticity



a. Viscoplastic potential



b. Plastic pseudo-potential as a limit case

- *Viscoplasticity* = after the choice of the function defining viscous effect, $\dot{v}$ is known
- *Plasticity* = $\dot{\lambda}$ to be defined from the consistency condition

Formulation of the plastic constitutive equations

- elastic domain : $f(\underset{\sim}{\sigma}, Y_I) < 0$ $\quad (\underset{\sim}{\dot{\epsilon}} = \underset{\approx}{\boldsymbol{\Lambda}}^{-1} : \underset{\sim}{\dot{\sigma}})$
- elastic unloading : $f(\underset{\sim}{\sigma}, Y_I) = 0$ and $\dot{f}(\underset{\sim}{\sigma}, Y_I) < 0$ $\quad (\underset{\sim}{\dot{\epsilon}} = \underset{\approx}{\boldsymbol{\Lambda}}^{-1} : \underset{\sim}{\dot{\sigma}})$
- plastic flow : $f(\underset{\sim}{\sigma}, Y_I) = 0$ and $\dot{f}(\underset{\sim}{\sigma}, Y_I) = 0$ $\quad (\underset{\sim}{\dot{\epsilon}} = \underset{\approx}{\boldsymbol{\Lambda}}^{-1} : \underset{\sim}{\dot{\sigma}} + \underset{\sim}{\dot{\epsilon}}^{p})$

$$\underset{\sim}{\dot{\epsilon}}^{p} = ...$$

$$\dot{Y}_I = ...$$

Flow directions associated with von Mises criterion

$f(\underset{\sim}{\boldsymbol{\sigma}}) = J(\underset{\sim}{\boldsymbol{\sigma}}) - \sigma_y$ (no hardening)

$$\underset{\sim}{\boldsymbol{n}} = \frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} = \frac{\partial J}{\partial \underset{\sim}{\boldsymbol{\sigma}}} = \frac{\partial J}{\partial \underset{\sim}{\boldsymbol{s}}} : \frac{\partial \underset{\sim}{\boldsymbol{s}}}{\partial \underset{\sim}{\boldsymbol{\sigma}}} \quad \text{where :} \quad n_{ij} = \frac{\partial J}{\partial s_{kl}} \frac{\partial s_{kl}}{\partial \sigma_{ij}}$$

$$\frac{\partial s_{kl}}{\partial \sigma_{ij}} = \delta_{ik}\,\delta_{jl} - \frac{1}{3}\,\delta_{ij}\,\delta_{kl}$$

$$n_{ij} = \frac{3}{2}\,\frac{s_{ij}}{J} \quad \text{where:} \quad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\,\frac{\underset{\sim}{\boldsymbol{s}}}{J}$$

Pure tension along direction 1 :

$$\underset{\sim}{\boldsymbol{s}} = \frac{2\,\sigma}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} \;;\; J = |\sigma| \;;\; \underset{\sim}{\boldsymbol{n}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} sign(\sigma)$$

Prandtl-Reuss law (1)

$$f(\underset{\sim}{\boldsymbol{\sigma}}, R) = J(\underset{\sim}{\boldsymbol{\sigma}}) - \sigma_y - R(p)$$

- Hardening curve for one-dimensional monotonic loading:
$\sigma = \sigma_y + R(p)$.
- Plastic modulus: $H = dR/d\epsilon^p = dR/dp$

For pure tension:

$$n_{11} = sign(\sigma) \quad , \quad n_{22} = n_{33} = (-1/2)n_{11}$$

$$\dot{\epsilon}^p_{11} = \dot{\epsilon}^p = sign(\sigma)\dot{\lambda} \quad , \quad \dot{\epsilon}_{22} = \dot{\epsilon}_{33} = (-1/2)\dot{\epsilon}^p$$

$$\dot{p} = |\dot{\epsilon}^p| = \dot{\lambda}$$

For general 3D case:

$$\underset{\sim}{\dot{\boldsymbol{\epsilon}}^p} : \underset{\sim}{\dot{\boldsymbol{\epsilon}}^p} = \dot{\lambda}^2 \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\dot{\lambda} \quad \text{then } \dot{p} = \left(\frac{2}{3}\underset{\sim}{\dot{\boldsymbol{\epsilon}}^p} : \underset{\sim}{\dot{\boldsymbol{\epsilon}}^p}\right)^{1/2}$$

Prandtl-Reuss law (2)

- Use of the consistency condition:

$$\frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} + \frac{\partial f}{\partial R}\dot{R} = 0 \quad \text{writes:} \quad \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} - H\dot{p} = 0 \quad \text{and:}$$

$$\dot{\lambda} = \frac{\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}}}{H} \text{ with } \qquad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{s}}}{J}$$

$$\dot{\underset{\sim}{\boldsymbol{\epsilon}}}^{p} = \dot{\lambda}\,\underset{\sim}{\boldsymbol{n}} = \frac{\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}}}{H}\,\underset{\sim}{\boldsymbol{n}}$$

For pure tension:

$$n_{11} = sign(\sigma) \quad , \quad \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} = \dot{\sigma}\, sign(\sigma) \quad \text{and:} \quad \dot{\lambda} = \dot{p} = \dot{\epsilon}^{p}_{11}$$

$$\text{so that :} \quad \dot{\epsilon}^{p} = \frac{n_{11}\,\dot{\sigma}}{H}\, n_{11} = \frac{\dot{\sigma}}{H}$$

Prager rule (1)

$$f(\underset{\sim}{\sigma}, \underset{\sim}{\boldsymbol{X}}) = J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}) - \sigma_y \quad \text{with} \quad J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}) = ((3/2)(\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}) : (\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}))^{0,5}$$

One-dimensional loading :

$$|\sigma - X| - \sigma_y = 0$$

Tensile curve modeled by:

$$\sigma = X(\epsilon^p) + \sigma_y$$

Since $\underset{\sim}{\boldsymbol{X}}$ is proportional to $\underset{\sim}{\epsilon}^p$, its components for one-dimensional loading are

$$X_{11}, X_{22} = X_{33} = -(1/2)X_{11}$$

Let us define:

$$\underset{\sim}{\boldsymbol{X}} = (2/3)H\underset{\sim}{\epsilon}^p$$

For one-dimensional loading, assume:

$$X = (3/2)X_{11} = H\epsilon_{11}^p$$

then

$$\begin{aligned} \underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}} &= diag\,((2/3)\sigma - X_{11}, -(1/3)\sigma + X_{11}/2, id) \\ &= diag\,((2/3)(\sigma - X), -(1/3)(\sigma - X), id)) \end{aligned}$$

$$J(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}) = |\sigma - X|$$

Prager rule (2)

Consistency condition:

$$\frac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} + \frac{\partial f}{\partial \underset{\sim}{\boldsymbol{X}}} : \dot{\underset{\sim}{\boldsymbol{X}}} = 0 \quad \text{then :} \quad \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} - \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{X}}} = 0 \quad \text{with :} \quad \underset{\sim}{\boldsymbol{n}} = \frac{3}{2} \frac{\underset{\sim}{\boldsymbol{s}} - \underset{\sim}{\boldsymbol{X}}}{J(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}})}$$

$$\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}} = \underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{X}}} = \underset{\sim}{\boldsymbol{n}} : \frac{2}{3} H \dot{\lambda} \underset{\sim}{\boldsymbol{n}} = H \dot{\lambda} \quad \text{so that :} \quad \dot{\lambda} = (\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}})/H$$

$$\dot{\underset{\sim}{\epsilon}}^p = \dot{\lambda} \underset{\sim}{\boldsymbol{n}} = \frac{\underset{\sim}{\boldsymbol{n}} : \dot{\underset{\sim}{\boldsymbol{\sigma}}}}{H} \underset{\sim}{\boldsymbol{n}}$$

- Same formal expression than for isotropic hardening, nevertheless $\underset{\sim}{\boldsymbol{n}}$ is different;
- Under one-dimensional loading, $\sigma = \sigma_{11}$, with $X = (3/2)X_{11}$:

$$|\sigma - X| = \sigma_y \quad , \quad \dot{\sigma} = \dot{X} = H\dot{\epsilon}^p$$

Summary in plasticity and viscoplasticity

For both cases:

- elastic domain defined by the load function $f < 0$;
- isotropic and kinematic hardenings

For plastic materials:

- plastic flow defined by the consistency condition, $f = 0$, $\dot{f} = 0$;
- plastic flow is *time independent* :

$$d\varepsilon^p = g(\sigma, \ldots) d\sigma$$

For viscoplastic materials:

- viscoplastic flow is defined by the value of the overstress $f > 0$;
- possible hardening on the viscous stress;
- viscoplastic flow if *time dependent* :

$$d\varepsilon^{vp} = g(\sigma, \ldots) dt$$

State variables

- Isotropic hardening depend on **p**, the *accumulated plastic strain* defined as :

$$\dot{p} = |\dot{\epsilon}^p|$$

- Linear kinematic hardening depend on ϵ^p , the *present plastic strain*

- Nonlinear kinematic hardening depend on α , defined as :

$$\dot{\alpha} = (\ 1 - D\ \alpha\ sign(\dot{\epsilon}^p)\ )\ \dot{\varepsilon}^p$$

asymptotic value of α *= 1 / D*

Isotropic/Kinematic hardening

$$f\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \underset{\sim}{\boldsymbol{X}}\right) - R$$

- **Non-linear isotropic hardening**

  $R = R0 + Q\ \left(1\ -\ e^{-bp}\right)$

  saturation rate: b , saturation hardening: Q

- **Non-linear kinematic hardening**

  $\underset{\sim}{\boldsymbol{X}} = \frac{2}{3}\, C\, \underset{\sim}{\alpha}$, $\dot{\underset{\sim}{\alpha}} = \dot{p}\left[\underset{\sim}{\boldsymbol{n}} - \frac{3D}{2C}\, \underset{\sim}{\boldsymbol{X}}\right]$

  saturation rate: D , saturation hardening: $\frac{C}{D}$

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Cyclic plasticity and hardening models

- kinematic hardening : shape of the stress-strain loops
- isotropic hardening :
  cyclic hardening ($Q > 0$) or softening ($Q < 0$)

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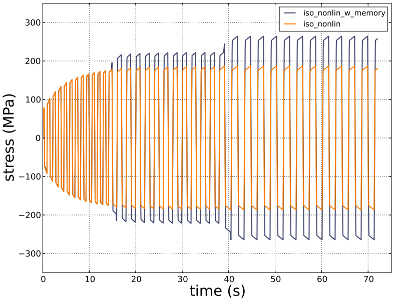
Strain-range memory effect

- Non linear isotropic hardening with strain-range memory:



$$Q = Q_0 + (Q_{sat} - Q_0)\exp(-2\mu q)$$
$$R = R_0 + Q\left(1 - e^{-\mathbf{b}p}\right)$$

$$\underset{\sim}{\boldsymbol{n}}^* = \tfrac{1}{2}(\underset{\sim}{\boldsymbol{\epsilon}}_{\boldsymbol{vi}} - \underset{\sim}{\boldsymbol{z}})/J(\underset{\sim}{\boldsymbol{\epsilon}}_{\boldsymbol{vi}} - \underset{\sim}{\boldsymbol{z}})$$
$$\eta = \underset{\sim}{\boldsymbol{n}} : \underset{\sim}{\boldsymbol{n}}^*$$
$$\dot{q} = \eta\dot{\lambda}$$

$$\dot{\underset{\sim}{\boldsymbol{z}}} = 2(\underset{\sim}{\boldsymbol{n}}^* : \underset{\sim}{\boldsymbol{\epsilon}}_{\boldsymbol{vi}})\underset{\sim}{\boldsymbol{n}}^*$$

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Multi-kinematic models

$$f\left(\underset{\sim}{\sigma}\right) = J\left(\underset{\sim}{\sigma} - \sum \underset{\sim}{\boldsymbol{X}}_i\right) - R$$

- smooth out the transition from linear to nonlinear behavior
- modelling of short and long-range hardening mechanisms
- for model calibration :
  - fix D values to scan the saturation rates :
    $D1 = 200$, $D2 = 4\ D1$, $D3 = 4\ D2$ etc...
  - find the C values using optimization

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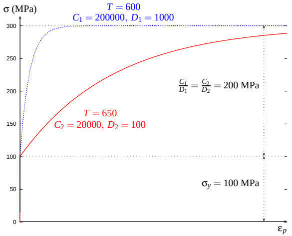
Temperature-dependency of kinematic coefficients

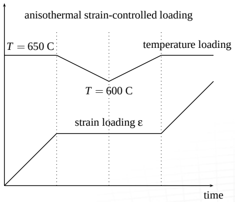
- purpose : allow a change in shape of the stress-strain loop when temperature changes during anisothermal applications

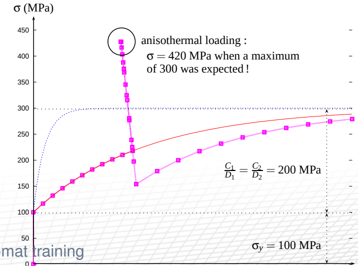


- yield function with kinematic hardening: $f(\underset{\sim}{\sigma}) = J(\underset{\sim}{\sigma} - \underset{\sim}{X}) - \sigma_y$
- back-stress: $\underset{\sim}{X} = \frac{2}{3}\, C(T)\, \underset{\sim}{\alpha}$
- $\dot{\underset{\sim}{\alpha}} = \dot{p}\, \left(\underset{\sim}{n} - D(T)\, \underset{\sim}{\alpha}\right)$
- C, D coefficients depending on temperature

should have $\sigma < 300$ MPa

wrong results may happen when D depends on temperature

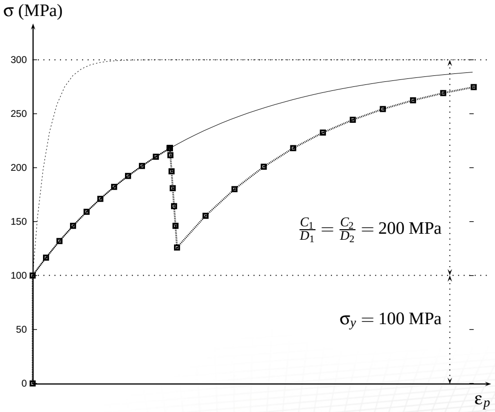




Patch for D temperature dependency (1/2)

- add a constraint $\left(J(\underset{\sim}{\alpha}) = \sqrt{\frac{2}{3}\underset{\sim}{\alpha} : \underset{\sim}{\alpha}} \right) \leq \frac{1}{D(T)}$
- modified evolution equation for $\underset{\sim}{\alpha}$

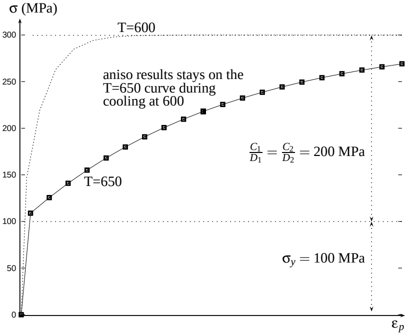
$$\dot{\underset{\sim}{\alpha}} = \dot{p}\,\left(\underset{\sim}{\boldsymbol{n}} - D(T)\,\underset{\sim}{\alpha}\right) - \left\langle J(\underset{\sim}{\alpha}) - \frac{1}{D(T)} \right\rangle \left(\frac{1}{J(\underset{\sim}{\alpha})} \; \frac{dD(T)}{dT} \; \frac{\dot{T}}{D^2} \right) \underset{\sim}{\alpha}$$




```
***behavior gen_evp
 ...
 **potential gen_evp ep
  *kinematic nonlinear
   aniso_correction
   C temperature
    200000.0 600.0
    20000.0 650.0
   D temperature
    1000.0 600.0
    100.0 650.0
 ..
***return
```

Patch for *D* temperature dependency (2/2)

- use several *X* with a constant *D* value for each




```
***behavior gen_evp
 ...
 **potential gen_evp ep
  *kinematic nonlinear
   C temperature
    200000.0 600.0
    0.0 650.0
   D 1000.0
  *kinematic nonlinear
   C temperature
    0.0 600.0
    20000.0 650.0
   D 100.0   ... ***return
```

- more refined patch than the previous *aniso_correction* one
- avoid any spurious relaxation during undercooling at 600 C
- difficulties if several *X* are needed to calibrate isothermal loops

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Asymmetric strain-controlled cyclic test

- linear kinematic hardening : non-zero mean-stress
- non-linear kinematic hardening : mean-stress relaxation to zero
- linear + non-linear : mean-stress relaxation to non-zero value

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Asymmetric stress-controlled cyclic test

- linear kinematic hardening : no ratcheting
- non-linear kinematic hardening :
  possible ratcheting with a constant strain increase
- linear + non-linear : arrest of the ratcheting effect

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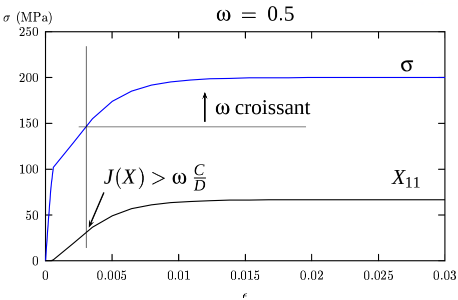
Non-linear kinematic with threshold



$$\underset{\sim}{X} = \frac{2}{3} C \underset{\sim}{\alpha}$$

$$\dot{\underset{\sim}{\alpha}} = \left[\underset{\sim}{n} - \frac{3D}{2C} \underset{\approx}{\Phi} : \underset{\sim}{X} \right] \dot{p}$$

$$\underset{\approx}{\Phi} = \left\langle \frac{D\, J(\underset{\sim}{X}) - \omega\, C}{1 - \omega} \right\rangle \frac{1}{(D\, J(\underset{\sim}{X}))} \underset{\approx}{I}$$

- switches from linear to nonlinear when $J(\underset{\sim}{X}) > \frac{C}{D}$
- refined modelling of mean-stress relaxation and ratcheting effects
- several objects may be needed to avoid slope discontinuities

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Viscoplasticity

Strain partition : $\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + (\underset{\sim}{\epsilon}^{th}) + \underset{\sim}{\epsilon}^{v}$

Normality rule : $\dot{\underset{\sim}{\epsilon}}^{v} = \dot{\lambda} \dfrac{\partial f}{\partial \underset{\sim}{\boldsymbol{\sigma}}}$

Norton flow law : $\dot{\lambda} = \left(\dfrac{\langle f(\underset{\sim}{\boldsymbol{\sigma}}) \rangle}{K} \right)^{n}$

- strain rate sensitivity
- creep
- stress relaxation

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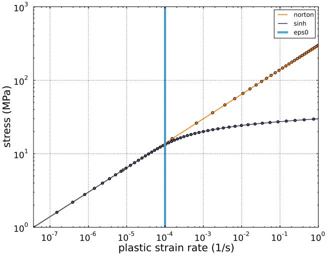
Norton vs Hyperbolic sine

- Hyperbolic sine function often used for more accurate correlation of strain rate sensitivity over a wide range of inelastic strain rates



$$\dot{p} = \left\langle \frac{\sigma_\upsilon}{K_n} \right\rangle^{n_n} \text{ - Norton power low}$$

$$\dot{p} = \epsilon_0 \left[\sinh\left(\left\langle \frac{\sigma_\upsilon}{K_h} \right\rangle^{n_h} \right) \right]^m \text{ - Hyperbolic law}$$

same response for both models for low plastic deformation rates

- Coefficient ϵ_0 defines strain rate at which hyperbolic law deviates from the classical norton response
- In order to adjust the coefficient K_h for $\sigma_\upsilon \rightarrow 0$ a simple rule can be applied:

suppose $n_n = n_h = n$ **and** $m = 1,$ **then** $K_h = \epsilon_0^{\frac{1}{n}} K_n$

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Aging effects

- time-dependent microstructure changes (unstable materials)
- activated at high temperature
- may produce softening (eg. aluminium alloys) or hardening (some steels)
- no dependency on prior strain deformation

- internal variable evolution accounting for aging effects

$$\dot{a} = \left\langle \frac{a_\infty(T) - a}{\tau(T)} \right\rangle \quad , \quad 0 \le a \le a_\infty$$

  assymptotic aging a_∞ and saturation rate τ depending on temperature

- influence of a on isotropic hardening (initial yield stress σ_y)

$$\sigma_y = R0 + R0^* \; (1 - a)$$

- influence of a on kinematic hardening

$$\underset{\sim}{\boldsymbol{X}} = \frac{2}{3} \, C \; (1 - a) \, \underset{\sim}{\boldsymbol{\alpha}}$$

```
***behavior gen_evp
 ...
 **potential gen_evp
 ...
 *kinematic nonlinear
  % C == (1.0-factor*age)*C
  C aging_effects param:age factor:1.0
  10000.0
  D 100.
 *isotropic isotropic_sum
  *-constant
   R0 100.0
  *-isotropic_aging
   R0_star 200.0
   a_inf 0.8
   tau 5000.0
 ...
***return
```

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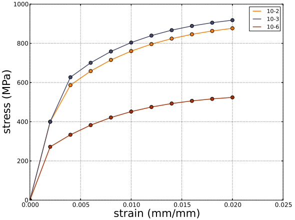
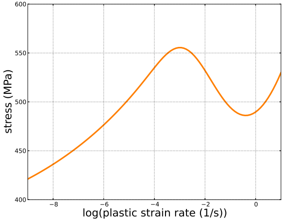
Inverse strain rate sensitivity (Portevin-Le Chatelier effect)

- Effect is observed for many metallic materials in some temperature and strain rate domains
- Associated with dynamic strain aging (DSA)



Inverse strain rate sensitivity



Evolutions with plastic strain rate of the normal viscous stress

$$R = R_{iso} + R_a \qquad R_a = P1\left(1 - e^{-\left(\frac{t_a}{t_0}\right)^{\beta}}\right) \qquad \dot{t}_a = 1 - \frac{t_a \dot{p}}{w}$$

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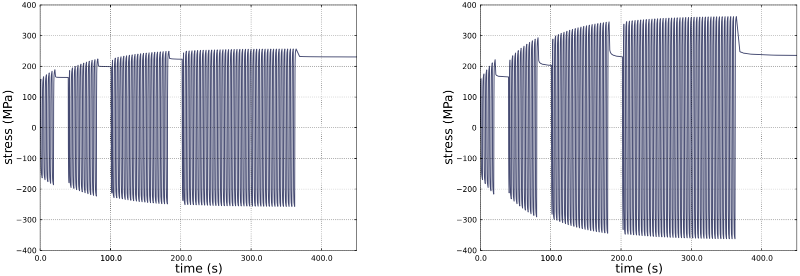
Marquis effect on K

- Influence of cyclic hardening on the viscous part of the stress

$$\sigma_v = K\dot{p}^{1/n} \qquad K = K_0 + \xi R$$



$\xi = 0$ $\qquad$ $\xi = 4$

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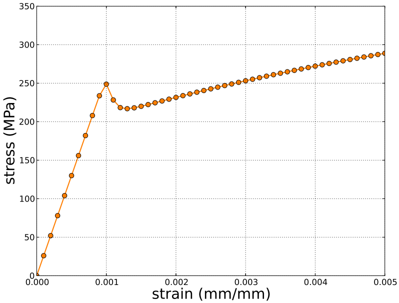
Static Strain Aging

- Same constitutive model for both SSA and DSA
- Possible application: simulation of the peak in the stress-strain curve around the yield strength



$$R = R_{iso} + R_a$$

$$R_a = P1\left(1 - e^{-\left(\frac{t_a}{t_0}\right)^{\beta}}\right)$$

$$\dot{t}_a = 1 - \frac{t_a \dot{p}}{w}$$

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Yield surface evolutions

- Z-sim option to draw yield surface evolutions

```
****simulate
 ***test creep
  **load *segment 5
    time sig11 sig22 sig33 sig12
    0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
    1.000000e-03 0.000000e+00 2.000000e+02 0.000000e+00 0.000000e+00
    1.000000e+03 0.000000e+00 2.000000e+02 0.000000e+00 0.000000e+00
  **model ...
  **yield_surface yield_11_22_0
   *degrees 5.0
   *factor 1000.0
   *find_offset
   *component sig11 sig22
   *time 1000.0
****return
```

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Kinematic hardening with static recovery

- time-dependent recovery of the hardening at high temperature

$$\dot{\underset{\sim}{\alpha}} = \dot{\lambda}\left[\underset{\sim}{\boldsymbol{n}} - \frac{3}{2}\frac{D}{C}\underset{\sim}{\boldsymbol{X}}\right] - \frac{3}{2}\frac{\underset{\sim}{\boldsymbol{X}}}{J(\underset{\sim}{\boldsymbol{X}})}\left(\frac{J(\underset{\sim}{\boldsymbol{X}})}{M}\right)^{m}$$

- constant strain rate during creep tests due to a balance of defect creations (hardening) and destruction (recovery)

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Multi potential models

Strain partition : $\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{el} + \left(\underset{\sim}{\epsilon}^{th}\right) + \sum \underset{\sim}{\epsilon}^{i}$

- refined modelling of a wide range of strain rates associated with different deformation mechanisms
- each potential can have its own flow law and hardening objects

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Interaction between potentials

Example: Two inelastic deformations, one plastic, the other one viscoplastic

Strain partition :

$$
\begin{aligned}
\underset{\sim}{\epsilon} &= \underset{\sim}{\epsilon}^{el} + \underset{\sim}{\epsilon}^{p} + \underset{\sim}{\epsilon}^{v} \\
f^p\left(\underset{\sim}{\boldsymbol{\sigma}}\right) &= J\left(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}^p\right) - R^p \\
f^v\left(\underset{\sim}{\boldsymbol{\sigma}}\right) &= J\left(\underset{\sim}{\boldsymbol{\sigma}} - \underset{\sim}{\boldsymbol{X}}^v\right) - R^v
\end{aligned}
$$

Hardening :

$$
\begin{aligned}
\underset{\sim}{\boldsymbol{X}}^p &= \tfrac{2}{3}\, C_p\, \underset{\sim}{\alpha}^p + C_{vp}\, \underset{\sim}{\alpha}^v \\
\dot{\underset{\sim}{\alpha}}^p &= \dot{p}\left[\underset{\sim}{\boldsymbol{n}}^p - \tfrac{3D}{2C}\, \underset{\sim}{\boldsymbol{X}}^p\right] \\
\underset{\sim}{\boldsymbol{X}}^v &= \tfrac{2}{3}\, C_v\, \underset{\sim}{\alpha}^v + C_{vp}\, \underset{\sim}{\alpha}^p \\
\dot{\underset{\sim}{\alpha}}^v &= \dot{\lambda}\left[\underset{\sim}{\boldsymbol{n}}^v - \tfrac{3D}{2C}\, \underset{\sim}{\boldsymbol{X}}^v\right]
\end{aligned}
$$

- alternative way to model inverse strain rate effect :
  Portevin-Le Chatelier effect in austenitic stainless steels

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2M1C model

- 2 mechanisms, 1 criterion
- allows to control the amount of ratcheting

Criterion : $f = \sqrt{J(\underset{\sim}{\sigma} - \underset{\sim}{X}_1)^2 + J(\underset{\sim}{\sigma} - \underset{\sim}{X}_2)^2} - R$

Kinematic hardening with coupling term :

$$\begin{aligned} \underset{\sim}{X}_1 &= \tfrac{2}{3}\left(C_{11}\,\underset{\sim}{\alpha}_1 + C_{12}\,\underset{\sim}{\alpha}_2\right) \\ \underset{\sim}{X}_2 &= \tfrac{2}{3}\left(C_{22}\,\underset{\sim}{\alpha}_2 + C_{12}\,\underset{\sim}{\alpha}_1\right) \end{aligned}$$

Kinematic evolution : $\dot{\underset{\sim}{\alpha}}_i = \dot{\lambda}\left(\underset{\sim}{n}_i - \frac{3D_i}{2C_{ii}}\,\underset{\sim}{X}_i\right)$

with : $\underset{\sim}{n}_i = \frac{3}{2}\frac{\underset{\sim}{s} - \underset{\sim}{X}_i}{J(\underset{\sim}{\sigma} - \underset{\sim}{X}_i)}$

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Addition of damage objects

Example: Viscoplastic damage (stage III creep)

Effective stress : $\underset{\sim}{\sigma} = (1-d)\, \underset{\approx}{\boldsymbol{D}}_{el} : \underset{\sim}{\epsilon}_{el} = (1-d)\, \underset{\sim}{\sigma}_{eff}$

Damage evolution using the Hayhurst function :

$$\dot{d} = \left\langle \frac{\chi(\underset{\sim}{\sigma})}{A} \right\rangle^{r} (1-d)^{-k}$$

$$\chi\left(\underset{\sim}{\sigma}\right) = \alpha\, \sigma_I + \beta\, tra(\underset{\sim}{\sigma}) + (1-\alpha-\beta) J$$

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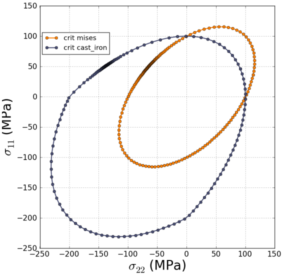
Cast Iron behavior

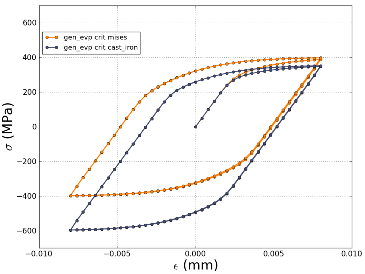
Ingredients of the cast iron model:

- Modified criterion:

$$f_t = \left(J^2 + (R_c - R_t)\,Tr\,\underset{\sim}{\sigma}\right)^{\frac{1}{2}} - (R_t R_c)^{\frac{1}{2}}$$

$$f_c = J - R_c$$





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Single-crystal model

Plastic deformation : $\dot{\underset{\sim}{\epsilon}}^p = \sum_r \dot{\gamma}^r \, \underset{\sim}{\boldsymbol{m}}^r$

for each slip system r : orientation tensor $\underset{\sim}{\boldsymbol{m}}^r$, slip γ^r

Resolved shear stress : $\tau^r = \underset{\sim}{\boldsymbol{\sigma}} : \underset{\sim}{\boldsymbol{m}}^r$

Viscoplastic flow : $\dot{v}^r = \left\langle \frac{f^r}{K} \right\rangle^n$, $\dot{\gamma}^r = \dot{v}^r \, sign(\tau^r - x)$

Criterion : $f^r = |\tau^r - x^r| - r^r - \tau_0$

Kinematic hardening : $x^r = C \, \alpha^r$

$\dot{\alpha}^r = (sign(\tau^r - x^r) - D \, \alpha^r) \, \dot{v}^r$

Isotropic hardening : $r^r = Q \sum_s h_{rs} \, (1 - exp(-b \, v^s))$

h_{rs} : interaction matrix

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Polycrystal model

- Micro-mechanical crystallographic model
- G grain phases defined by their volume fraction f_g and orientation
- Localization rule for stress $\underset{\sim}{\sigma}_g$ in each grain :

$$\underset{\sim}{\sigma}_g = \underset{\sim}{\Sigma} + C \left[\sum_{i \in G} (f_i \beta_i) - \underset{\sim}{\beta}_g \right]$$

- Evolution of inter-granular hardening tensors $\underset{\sim}{\beta}_g$:

$$\underset{\sim}{\dot{\beta}}_g = \underset{\sim}{\dot{\epsilon}}^p_g - D\, \underset{\sim}{\beta}_g\, ||\underset{\sim}{\dot{\epsilon}}^p_g||$$

- Homogenization of plastic strains : $\underset{\sim}{\dot{E}}^p = \sum_g f_g\, \underset{\sim}{\dot{\epsilon}}^p_g$
- Single-crystal constitutive equations for each phase g

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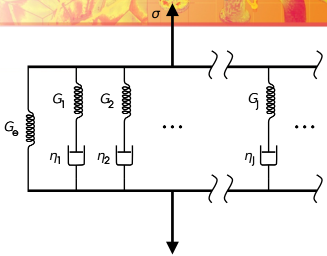
Viscoelastic generalized Maxwell model (1/2)

Strain partition : $\underset{\sim}{\epsilon} = \underset{\sim}{\epsilon}^{vel} + (\underset{\sim}{\epsilon}^{th}) + \underset{\sim}{\epsilon}^{vp}$

Viscoelastic deformation is described by the Maxwell model



The model defines the stress, $\underset{\sim}{\sigma}$, to the strain $\underset{\sim}{\epsilon}$ by the following relation:

$$\underset{\sim}{\boldsymbol{\sigma}}(\boldsymbol{t}) = \int_0^t 2G(t-\tau)Dev(\dot{\underset{\sim}{\epsilon}})(\tau)d\tau + \underset{\sim}{\boldsymbol{I}} \int_0^t K(t-\tau)Tr(\dot{\underset{\sim}{\epsilon}})d\tau \qquad (1)$$

The terms G and K are relaxation functions defined by Prony series:

$$G(\tau) = G_\infty - (G_\infty - G_0)\Psi_1(\tau), \quad \Psi_1(\tau) = \sum_{i=1}^{i=n_\alpha} \gamma_i \exp(-\tau/\tau_i)$$

$$K(\tau) = K_\infty - (K_\infty - K_0)\Psi_2(\tau), \quad \Psi_2(\tau) = \sum_{i=1}^{i=n_\beta} \gamma_i \exp(-\tau/\tau_i) \qquad (2)$$

Viscoelastic generalized Maxwell model (2/2)

Coefficients:

- K_0 and K_∞ - instantaneous and long term bulk moduli ;
- G_0 and G_∞ - instantaneous and long term shear moduli ;
- τ_i - characteristic timescale of the i-th chain (Maxwell time) ;
- γ_i - shear or volumic relaxation modulus ratio of the i-th chain .

Features:

- strain rate sensitivity
- shear and volumic viscous effects
- creep
- elastic stress relaxation with a spread of relaxation times

Applications:

- polymers, metals at high temperature

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